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DEFORMATION OF A POROELASTIC HALF-SPACE DUE TO COMPENSATED LINEAR VECTOR DIPOLE

Heena Verma[*1], Ram Swaroop[2] & Vinay Kumar[3]
[*1]Department of Mathematics, NIMS University, Jaipur, India
[2]Department of Mathematics, NIMS University, Jaipur, India
[3]Department of CSE, Chandigarh University, Chandigarh, India

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ABSTRACT

The closed-form analytical expressions for the displacements and strains due to Compensated Linear Vector Dipole (CLVD) located in a homogeneous, isotropic poroelastic half-space are obtained. The variation of radial displacement and strains with depth & variation of radial displacement and strains with epicentral distance for the various materials Ruhr Sandstone, Tennessee Marble, Charcoal Granite, Berea Sandstone and Westerly Granite are discussed.

Keywords: Poroelastic half-space, displacement, epicentral distance, Compensated Linear Vector Dipole.

I. INTRODUCTION

A moment tensor is usually diagonalized and decomposed into some elementary parts. It can be decomposed into its isotropic (ISO) and deviatoric (DEV) parts. The double-couple (DC) and CLVD components often are described collectively as the deviatoric component. The total moment-tensor solution consists of an addition of the isotropic, double-couple, and CLVD components. The deviatoric part can be decomposed into three double couples by Jost and Herrmann (1989), into major and minor double couples by Kanamori and Given (1981), Wallace (1985) or into a double couple and a compensated linear vector dipole (CLVD) component given by Knopoff and Randall (1970).

The most common type of the moment tensor is the double-couple (DC) source which represents the force equivalent of shear faulting on a planar fault in isotropic media. However, many studies reveal that seismic sources often display more general moment tensors with significant non-double-couple (non-DC) components given by Julian et al. (1998), Miller et al. (1998). An explosion is an obvious example of a non-DC source, but non-DC components can also be produced by a collapse of a cavity in mines by Rudajev and Šílený (1985), by inflation or deflation of magma chambers in volcanic areas given by Mori and McKee (1987), by shear faulting on a nonplanar (curved or irregular) fault, by tensile faulting induced by fluid injection when the slip vector is inclined from the fault and causes its opening by Vavryčuk (2001, 2011), or by shear faulting in anisotropic media given by Kawasaki and Tanimoto (1981), Vavryčuk (2005).

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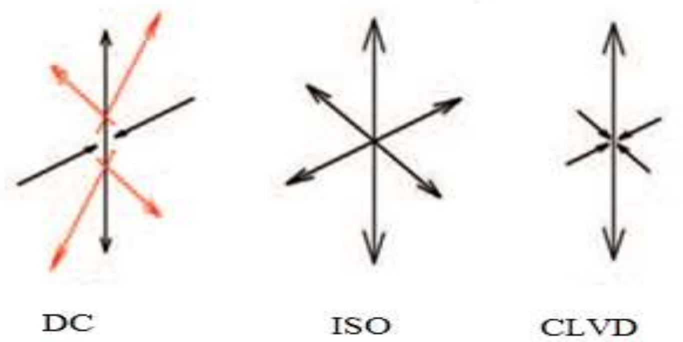
A purely volumetric source is known as an isotropic source (ISO) and is described by a moment tensor that contains equal-valued diagonal elements and zeroes for the off-diagonal elements given by Aki and Richards (2002). This describes the situation when one dipole is compensated by the two other dipoles, which are half the magnitude, i.e., the diagonal elements have a ratio of –1: –1: 2, whereas the off-diagonal elements are zero given by Julian et al. (1998) .Note that a trace of M = 0 indicates absence of volumetric changes in the source; hence CLVD and double-couple sources have no volumetric component (i.e., no dilation).

Care must be taken when interpreting a mechanism described as CLVD by the moment tensor because this also can be described by other possible mechanisms. For example, a CLVD mechanism can be created by two double-couple geometries with different moments of M0 and 2M0. CLVD components also can help to describe the opening or closing of a crack, along with an isotropic component given by Julian et al. (1998), in which the diagonal elements form a ratio of 1:1:3 for a Poisson's ratio of 0.25 and 1:1:2 for a Poisson's ratio of 0.35.

Figure 1:



Double couple, isotropic and CLVD sources.

arrows at the top schematically show the corresponding force systems

generating the source mechanisms (black) and the shear forces (red).

II. THEORY

A compensated linear vector dipole (Fig. 6.2), Knopoff and Randall (1970) consists of three mutually orthogonal dipoles with moments in the ratio (-1, -1, 2).

Figure 2:

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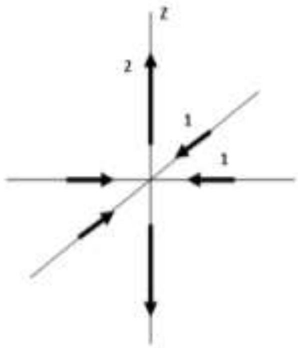

Fig. 6.2: Body forces equivalent to a compensated linear vector dipole

Displacements

Using displacement field given by Amit Kumar *et al.* (2012), we obtained the expressions for the displacement components due to Tensile Dislocation of magnitude **P** acting at the point (0, 0, c) in poroelastic half-space.
The displacements for the compensated linear vector dipole are obtained on using the relations:

$$u_i = -\frac{\partial u_i^1}{\partial x_1} - \frac{\partial u_i^2}{\partial x_2} + 2\frac{\partial u_i^3}{\partial x_3}$$

$$where \;\; i = 1,2,3$$

Where

$$u_1 = \frac{Px_1}{16\pi\mu}\left[4\left(\frac{1}{R_1^3} + \frac{(6(1-\tilde{\sigma})-1)}{R_2^3} - \frac{3(x_3+c)(3(x_3-c)+2c)}{R_2^5} \right) + \bar{\sigma}\left\{ \frac{1}{R_1^3} - \frac{1}{R_2^3} - \frac{9(x_3-c)^2}{R_1^5} + \frac{6cx_3}{R_2^5} + 3(x_3+c)^2\left(\frac{3}{R_2^5} - \frac{10cx_3}{R_2^7} \right) \right\} \right] \quad (1)$$

u_2 can be obtained by replacing x_1 to x_2 in eqn. (5)

$$u_3=\frac{P}{16\pi\mu}\left[\begin{array}{l}\left(\begin{array}{l}-4\left\{\frac{2(x_3-c)}{R_1^3}+\frac{4\tilde{\sigma}\,x_3}{R_2^3}+\frac{(9(x_3-c)+8c)(x_3+c)^2}{R_2^5}\right\}\\+\frac{(6c+(x_3-c))}{R_2^3}\end{array}\right)\\+\bar{\sigma}\left\{\begin{array}{l}(x_3-c)\left(\frac{7}{R_1^3}-\frac{9(x_3-c)^2}{R_1^5}\right)\\+(x_3+c)\left\{\frac{18c\,x_3}{R_2^5}+(x_3+c)\left(\frac{16\tilde{\sigma}\,c+(9(x_3-c)+8c)}{R_2^5}-\frac{30c\,x_3(x_3+c)}{R_2^7}\right)\right\}\end{array}\right\}\end{array}\right] \quad (2)$$

$$R_1^2=x_1^2+x_2^2+(x_3-c)^2\ ;\quad R_2^2=x_1^2+x_2^2+(x_3+c)^2 \quad (3)$$

$\kappa=\lambda+\frac{2}{3}\mu$ is bulk-modulus; λ , μ are Lamé constants;

$\hat{\sigma}=\frac{1-2\tilde{\sigma}}{1-\tilde{\sigma}};\bar{\sigma}=\frac{1}{1-\tilde{\sigma}};\tilde{\sigma}=\frac{\tilde{\lambda}}{2(\tilde{\lambda}+\mu)}$ is Poisson ratio. If the porosity disappears, then $\tilde{\lambda}\to\lambda$

Case 1: when z ≠ 0

Using $x_1=r\cos\theta;\quad x_2=r\sin\theta;\quad x_3=z$ in eqn (7) we get

$R_1^2=r^2+(z-c)^2,\qquad R_2^2=r^2+(z+c)^2$

The displacements components in cylindrical co-ordinates are given as

$u_r=u_1\cos\theta+u_2\sin\theta$; $u_\theta=u_1\sin\theta-u_2\cos\theta$; $u_z=u_3$

$$u_r=\frac{\Pr}{16\pi\mu}\left[\begin{array}{l}4\left(\begin{array}{l}\frac{1}{R_1^3}+\frac{(6(1-\tilde{\sigma})-1)}{R_2^3}\\-\frac{3(z+c)(3(z-c)+2c)}{R_2^5}\end{array}\right)\\+\bar{\sigma}\left\{\begin{array}{l}\frac{1}{R_1^3}-\frac{1}{R_2^3}-\frac{9(z-c)^2}{R_1^5}+\frac{6cz}{R_2^5}+\\3(z+c)^2\left(\frac{3}{R_2^5}-\frac{10cz}{R_2^7}\right)\end{array}\right\}\end{array}\right] \quad (4)$$

$$u_\theta=0 \quad (5)$$

$$u_z=\frac{P}{16\pi\mu}\left[\begin{array}{l}\left(\begin{array}{l}-4\left\{\frac{2(z-c)}{R_1^3}+\frac{4\tilde{\sigma}z}{R_2^3}+\frac{(9(z-c)+8c)(z+c)^2}{R_2^5}\right\}+\\\frac{(6c+(z-c))}{R_2^3}\end{array}\right)\\+\bar{\sigma}\left\{\begin{array}{l}(z-c)\left(\frac{7}{R_1^3}-\frac{9(z-c)^2}{R_1^5}\right)+\\(z+c)\left\{\frac{18cz}{R_2^5}+(z+c)\left(\frac{16\tilde{\sigma}c+(9(z-c)+8c)}{R_2^5}-\frac{30cz(z+c)}{R_2^7}\right)\right\}\end{array}\right\}\end{array}\right] \quad (6)$$

Strains

Strains in cylindrical co-ordinates can be calculated using $e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

Using equations (4)- (6) in strain displacement relations, we obtain the following expressions of strains due to a compensated linear vector dipole in a poroelastic medium are obtained as

$$e_{rr} = \frac{P}{16\pi\mu}\left[\begin{array}{l} 4\left\{\begin{array}{l} \frac{1}{R_1^3} + \frac{(6(1-\tilde{\sigma})-1)}{R_2^3} + \\ 3(z+c)(3(z-c)+2c)\left(\frac{5r^2}{R_2^7} - \frac{1}{R_2^5}\right) \\ + \frac{15r^2(z+c)(3(z-c)+2c)}{R_2^7} \end{array}\right\} \\ + \tilde{\sigma}\left\{\begin{array}{l} -\left(\begin{array}{l} \frac{8}{R_1^3} + \frac{1}{R_2^3} + \\ \frac{4r^2(z-c)^2}{R_1^7} + \frac{24cz}{R_2^5} \end{array}\right) \\ + 3(z+c)^2\left(\frac{3}{R_2^5} + 5r^2\left(\frac{14cz}{R_2^9} - \frac{3}{R_2^7}\right)\right) \end{array}\right\} + \frac{6r^2\tilde{\sigma}}{R_1^5} \end{array}\right] \tag{7}$$

$$e_{zz} = \frac{P}{16\mu\pi}\left[\begin{array}{l} 4\left\{\begin{array}{l} \frac{-2}{R_1^3} + \frac{2(8\tilde{\sigma}(1-2\tilde{\sigma})-(1+\tilde{\sigma}))}{R_2^3} \\ -3(z+c)\left(\begin{array}{l} \frac{4(1-\tilde{\sigma})(z-c)}{R_2^5} \\ -\frac{5(z+c)^2(3(z+c)-2c)}{R_2^7} \end{array}\right) \end{array}\right\} \\ + \tilde{\sigma}\left\{\begin{array}{l} \frac{7}{R_1^3} + \frac{9}{R_2^3} - 3(z-c)^2\left(\frac{8(1+\tilde{\sigma})}{R_1^5} - \frac{15(z-c)^2}{R_1^7}\right) \\ + \frac{18cz - 12(z+c)(z-c)\tilde{\sigma}}{R_2^5} \\ + 12(z+c)^2\left(\frac{2\tilde{\sigma}(1+\tilde{\sigma})}{R_2^5} - \frac{5c(4(z+c)-3c)}{R_2^7}\right) \\ - 15(z+c)^4\left(\frac{3}{R_2^7} + \frac{14cz}{R_2^9}\right) \end{array}\right\} \end{array}\right] \tag{8}$$

$$e_{rz}=\frac{\Pr}{16\pi\mu}\left[\left(6\left\{\begin{matrix}(z-c)\left(\frac{1}{R_1^5}-\frac{(3-4\tilde{\sigma})}{R_2^5}\right)+\frac{2\tilde{\sigma}z(1+4\tilde{\sigma})}{R_2^5}\\-2(z+c)\left(\frac{2(1-\tilde{\sigma})}{R_2^5}+\frac{15z(z+c)}{R_2^7}\right)\end{matrix}\right\}\right)\\ \bar{\sigma}\left\{\begin{matrix}3(z-c)\left(\frac{-7}{R_1^5}+\frac{15(z-c)^2}{R_1^7}-\frac{3}{R_2^5}\right)\\+2(z+c)\left(\frac{-3(1-2\tilde{\sigma})}{R_2^5}+\frac{5c(1-9z)}{R_2^7}\right)\\+15(z+c)^3\left(\frac{-3}{R_2^7}+\frac{16cz}{R_2^9}\right)\end{matrix}\right\}\right] \quad (9)$$

$$e_{zr}=e_{rz}$$
$$e_{\theta z}=0,\ e_{z\theta}=0$$
$$e_{\theta r}=0,\ e_{r\theta}=0 \quad (10)$$

Case 2: when z = 0

Displacements components are given as

$$u_r=\frac{3\Pr}{4\pi\mu}\left(\frac{2(1-\tilde{\sigma})}{R^3}+\frac{c^2}{R^5}\right) \quad (11)$$

$$u_z=\frac{Pc}{4\pi\mu}\left(\frac{-2[(1-3\tilde{\sigma})+\bar{\sigma}]}{R^3}+\frac{3(1+\tilde{\sigma})\bar{\sigma}c^2}{R^5}\right) \quad (12)$$

$$where\ \ R=\sqrt{r^2+c^2}$$

$$e_{rr}=\frac{P}{16\pi\mu}\left[\frac{-3(4(\tilde{\sigma}-2)+\bar{\sigma})}{R^3}+\frac{3c^2(6\tilde{\sigma}\bar{\sigma}-\hat{\sigma})}{R^5}-\frac{r^2c^2(60+49\bar{\sigma})}{R^7}\right] \quad (13)$$

$$e_{zz}=\frac{P}{16\pi\mu}\left[\frac{8\tilde{\sigma}(\tilde{\sigma}+1)(\tilde{\sigma}-16)}{R^3}+\frac{12c^2(\tilde{\sigma}-2)(\bar{\sigma}+2)}{R^5}-\frac{60\tilde{\sigma}\bar{\sigma}c^4}{R^7}\right] \quad (14)$$

Numerical Results

We define dimensionless epicentral distance D, dimensionless radial displacement U, dimensionless vertical displacement W (uplift) and dimensionless radial strain E by the relations

$$D=\frac{r}{c},U=\frac{Q'}{c}u_r,W=\frac{Q'}{c}u_z,E=Q'e_{rr}$$

Where Q' is a dimensionless constant for each source, chosen in such a manner that W=1 at r=0.

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$$U = \frac{-3D}{\left((1-6\tilde{\sigma}) - 4\tilde{\sigma}\bar{\sigma}\right)}\left[\frac{(3-2\tilde{\sigma}) + 2D^2(1-\tilde{\sigma})}{(1+D^2)^{5/2}}\right] \tag{15}$$

$$W = \frac{\bar{\sigma}}{\left((1-6\tilde{\sigma}) - 4\tilde{\sigma}\bar{\sigma}\right)}\left[\frac{2(\bar{\sigma} + (1-3\tilde{\sigma}))}{(1+D^2)^{3/2}} - \frac{3(1+\tilde{\sigma})\bar{\sigma}}{(1+D^2)^{5/2}}\right] \tag{16}$$

$$E = \frac{-\bar{\sigma}}{4\left((1-6\tilde{\sigma}) - 4\tilde{\sigma}\bar{\sigma}\right)}\left[\frac{\left((1-2\tilde{\sigma})(5-4\tilde{\sigma}) - 6\tilde{\sigma}^2\right)}{(1+D^2)^{3/2}} - \frac{3(1-8\tilde{\sigma})}{(1+D^2)^{5/2}} - \frac{(60(1-\tilde{\sigma}) + 49)D^2}{(1+D^2)^{7/2}}\right] \tag{17}$$

$$Q' = -\frac{4\pi\mu c^3}{P\left((1-6\tilde{\sigma}) - 4\tilde{\sigma}\bar{\sigma}\right)} \tag{18}$$

III. DISCUSSION AND CONCLUSION

Analytical expressions for the displacement and strain components due to five materials namely, Ruhr Sandstone, Tennessee Marble, Charcoal Granite, Berea Sandstone, Westerly Granite for drained behaviour for compensated linear vector dipole in a poroelastic medium has been obtained.

For analysis taking P=1, c=1, z=1, $\mu = \frac{3\kappa(1-2\tilde{\sigma})}{2(1+\tilde{\sigma})}$ and using Table 1

Table:

Table 1. Material property

S No	Materials	Poisson Ratio $(\tilde{\sigma})$	$\kappa(N/m^2)$
1	Ruhr Sandstone(RS)	0.12	1.3×10^{10}
2	Tennessee Marble(TM)	0.25	4.0×10^{10}
3	Charcoal Granite(CG)	0.27	3.5×10^{10}
4	Berea Sandstone(BS)	0.20	8.0×10^{9}
5	Westerly Granite(WG)	0.25	2.5×10^{10}

Fig 3 shows the variation of uplift with depth for drained behaviour of five materials i.e. RS, CG, TM, BS, WG. It shows that vertical displacement first increases, then decreases, follow the zigzag path and finally decreases. Decrease is more rapidly in case of BS as compared to TM.

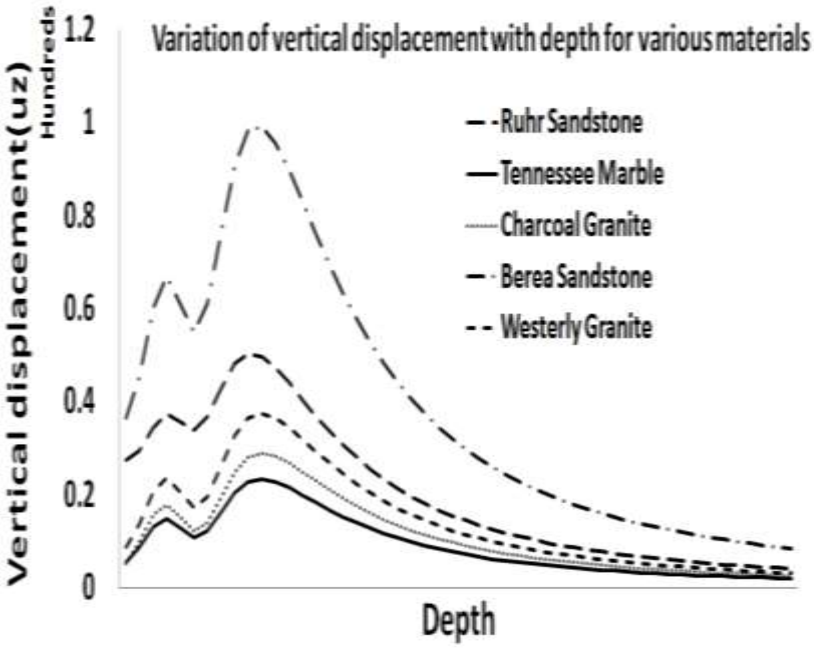
Fig 4 shows the variation of radial displacement with depth for drained behaviour of five materials i.e. RS, CG, TM, BS, WG. It shows that radial displacement decreases more rapidly in case of BS than as compared to TM .Hence variation for TM is less.

Fig 5 shows the variation of dimensionless radial displacement with epicentral distance For all these materials we observe that as we move away from epicentre the displacement decrease gradually. The rate of decrease is more in case of RS as compared to CG.

Fig 6 shows the variation of dimensionless vertical displacement (uplift) with epicentral distance. For all these materials, variations in vertical displacement (uplift) vary with the material. We observe that as we move away from epicentre the displacement decrease gradually.

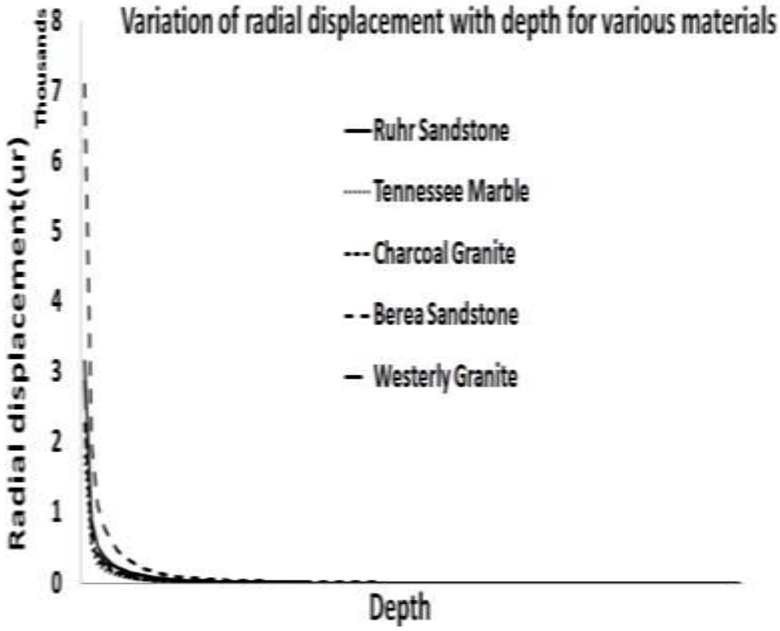
Fig 7 shows the variation of dimensionless radial strain with epicentral distance Rate of decrease is more in case of RS as compared to CG. Therefore RS shows more Variation as compared to CG .Value of poisson ratio is same for TM & WG.

Figure 3:



Variation of vertical displacement i.e. uplift with depth

Figure 4:



Variation of radial displacement with depth

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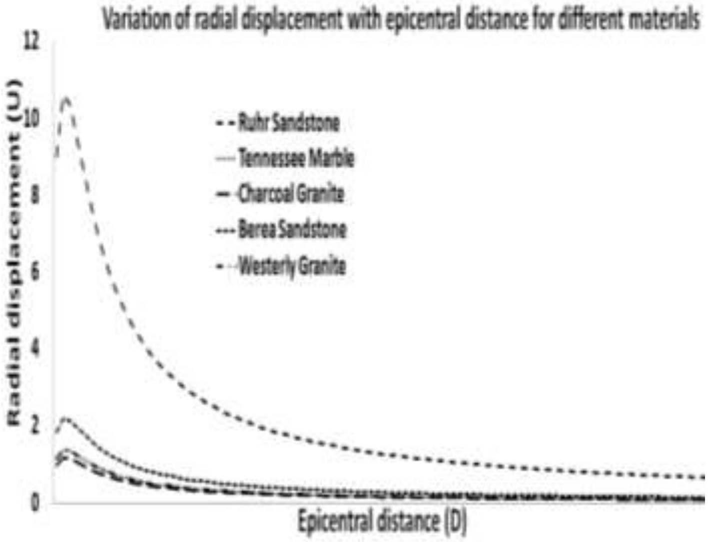
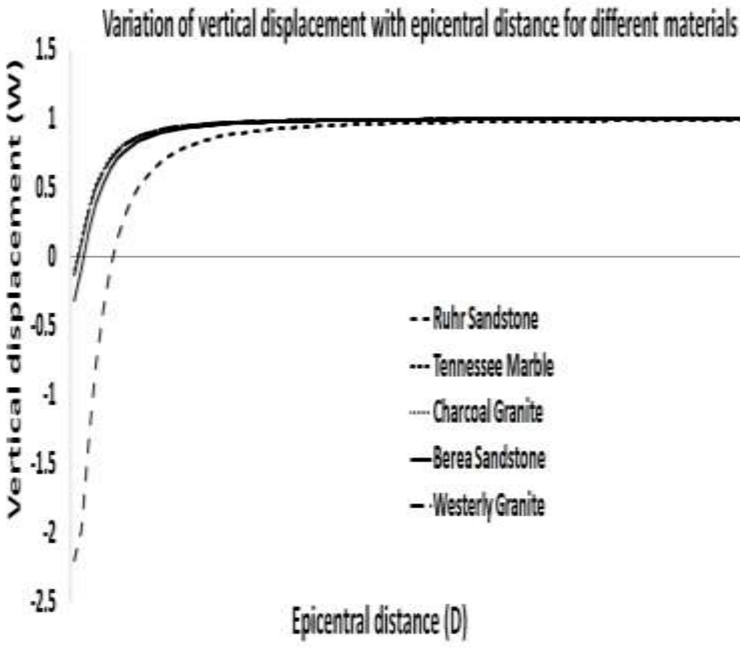
Figure 5:



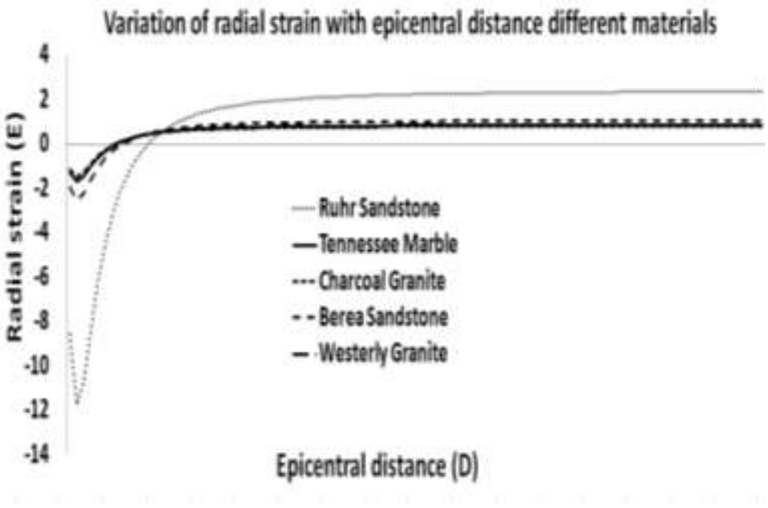
Variation of radial displacement with epicentral distance

Figure 6:



Variation of vertical displacement with epicentral distance

Figure 7:



Variation of radial strain with epicentral distance

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